



II Semester M.Sc. Examination, July 2017
(CBCS)

MATHEMATICS

M208 SC : Mathematical Modelling and Numerical Analysis - I

Time : 3 Hours

Max. Marks : 70

*Instructions : Answer any five full questions.
All questions carry equal marks.*

1. a) Explain briefly the concept of mathematical modelling. Discuss the steps and limitations involved in mathematical modelling.
b) Discuss the mathematical model to understand
 - i) Drug absorption from blood stream and
 - ii) Motion of projectile in the absence of air resistance. (8+6)

2. a) Discuss the mathematical model for the spread of epidemics.
b) Derive the one-dimensional wave equation for the vibration of a string without damping. (7+7)

3. a) Obtain the modified Newton-Raphson method that converges quadratically to find a multiple root of $f(x) = 0$ whose multiplicity is known in advance. Hence, find the double root of $x^3 - x^2 - x + 1 = 0$ with $x_0 = 0.8$.
b) Find the smallest root of $3x - \cos x - 1 = 0$ correct to four decimals using Ramanujan's method. (7+7)

4. a) Formulate the Aitken Δ^2 -method to accelerate the convergence of linear iteration method for finding the root of $f(x) = 0$ and hence find the root of $2x = \cos x + 3$.
b) Obtain the number of real and complex roots of $x^4 - 3x^2 + x - 2 = 0$ using Sturm sequences. (7+7)

5. a) Use the Doolittle method to solve the system :
 $x_1 + x_2 + x_3 = 6 ; 3x_1 + 3x_2 + 4x_3 = 21 ; 2x_1 + x_2 + 3x_3 = 13$.
b) Determine condition number of the matrix

$$\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}$$

Is the Hilbert matrix of order three is ill-conditioned ? Justify your answer. (7+7)

P.T.O.



6. a) Obtain the solution of the system :

$$x_1 + 2x_2 = 5, 2x_1 + 3x_2 - x_3 = 5, 4x_1 + 2x_3 + 3x_4 = 26, 2x_3 - 4x_4 = -10 \text{ using the Thomas algorithm.}$$

b) Solve the non-linear system of equations using the Newton-Raphson method :

$$x^2 - y^2 = 4, x^2 + y^2 = 16. \text{ Take } x_0 = 2\sqrt{2} = y_0 \text{ and perform two iterations. (7+7)}$$

7. a) Establish the Hermite interpolating polynomial in its standard form.

b) Obtain the natural cubic spline that interpolates the points.

$$(1, 1), (2, 1/2), (3, 1/3) \text{ and } (4, 1/4). \quad (7+7)$$

8. a) Obtain the rational approximation $R_{2,2}$ for the function $\cos \sqrt{x}$.

b) Establish the Gauss-Chebyshev three-point quadrature formula and hence

$$\text{evaluate } \int_{-1}^1 (1-x^2)^{1/2} \cos x \, dx. \quad (7+7)$$

BMSCW